IMPT NOTES FOR MATH220

- NAME: ARAVINTH
- 2nd Feb ← EXAM 1: 15%
- 2nd March ← EXAM 2: 15%
- 6th April ← EXAM 3: 15%
- RECITATION GRADE: 5%
- WHW: 15%
- OHW: 10%
- May 10 ← FINALS: 25%

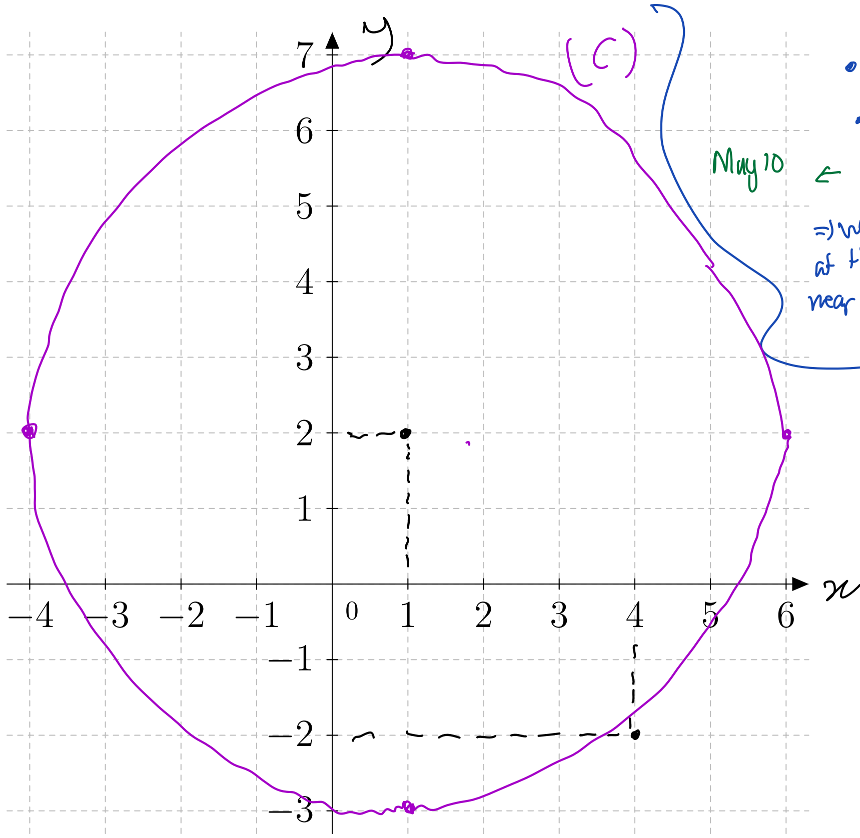
⇒ Written HW to be submitted at the homework box near cardwell 120

Math 220 – Worksheet 1

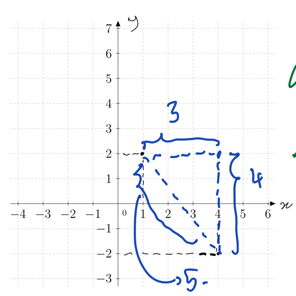
1. A. Plot the points $(1, 2)$ and $(4, -2)$ on the graph below.

(1, 2)
x-axis → Horizontal
y-axis → Vertical.

(4, -2)



B. Find the distance between $(1, 2)$ and $(4, -2)$. (Hint: One can use the Pythagorean Theorem.)



$a^2 = b^2 + c^2$

(Pythagorean theorem) tells us that given 2 coordinates (x_1, y_1) and (x_2, y_2), the distance between the 2 points are given by

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

- In this case, $(x_1, y_1) = (1, 2)$ and (x_2, y_2) is $(4, -2)$. So,

$\sqrt{(4-1)^2 + (-2-2)^2} = \sqrt{3^2 + (-4)^2} = \sqrt{25} = 5.$

C. Find the equation for all points 5 units away from the point $(1, 2)$. Sketch the set of these points in the graph above.

Notes on function transformation:

1. $f(x) + d$ ⇒ Vertical translation up by d units
2. $f(x) - d$ ⇒ Vertical translation down by d units
3. $f(x + c)$ ⇒ Horizontal translation left c units.
4. $f(x - c)$ ⇒ Horizontal translation right c units.

(5) $-f(x) \Rightarrow$ Reflection over x-axis.

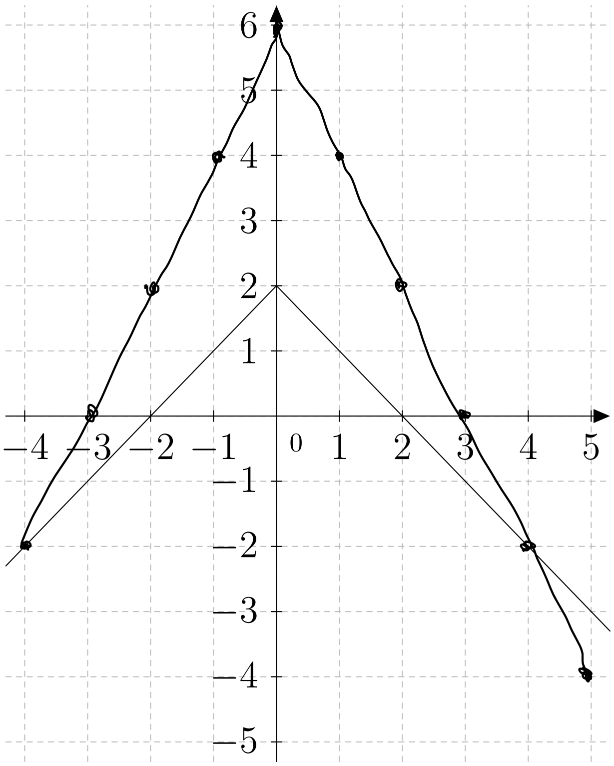
(6) $f(-x) \Rightarrow$ Reflection over y-axis.

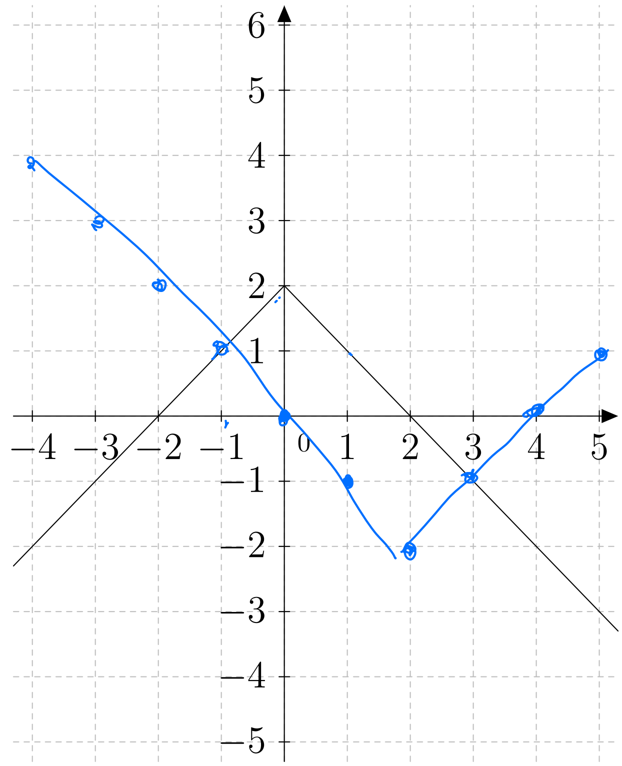
(7) $af(x) \Rightarrow \begin{cases} \text{vertical stretch if } |a|>1. \\ \text{Vertical compression for } 0<|a|<1. \end{cases}$

2. In the plots below, $y = f(x)$ is graphed. On the left, plot the graph of $y = 2f(x) + 2$. On the right, plot the graph of $y = -f(x-2)$.

(8) $f(bx) \begin{cases} \text{Horizontal compression for } |b|>1 \\ \text{Horizontal stretch for } 0<|b|<1 \end{cases}$

↳ $f(x) \xrightarrow{1} 2f(x) \xrightarrow{2} 2f(x)+2.$



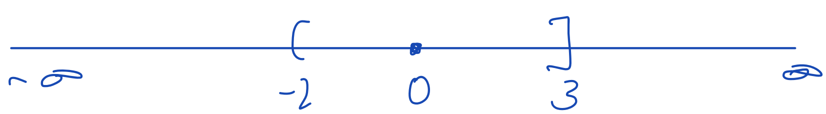


3. Describe in words the following sets of numbers.

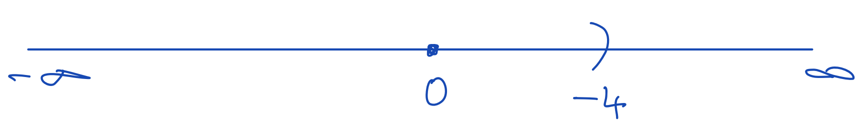
• Square bracket [or] means we include this point.

• Curved bracket (or) means, we exclude the point.

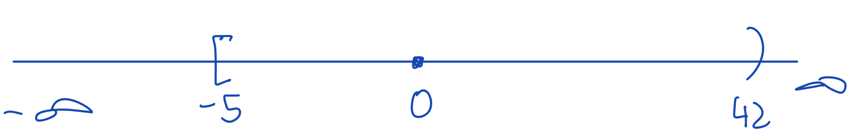
A. $(-2, 3]$



B. $(-\infty, 4)$



C. $-5 \le x < 42$



↳ x greater than or equal to −5

x lesser than 42.

4. Find the equation of the line that goes through $(1,1)$ and $(3,7)$.

Equation for a line given 2 points, (x_1, y_1) and (x_2, y_2): $(y-y_1) = m(x-x_1)$, where m is the gradient, $m = \frac{y_2-y_1}{x_2-x_1}$ $\Big\}$ So, we use the 2 coordinates given (x_1, y_1) and (x_2, y_2) to find the gradient.

Let $(x_1, y_1) = (1,1)$ and (x_2, y_2) be $(3,7)$. Then, $m = \frac{y_2-y_1}{x_2-x_1} = \frac{7-1}{3-1} = \frac{6}{2} = 3$.

Then, plugging in the respective values onto the equation $y-y_1 = m(x-x_1)$, we get $y-1 = 3(x-1)$
$\Rightarrow y-1 = 3x-3$
$\Rightarrow y = 3x-2$.

5. Find the equation of the line with slope 4 that goes through $(-3,5)$.

In this case, unlike question 4, we are given a point and the slope.
the slope is the gradient, so we can skip the process of finding m.
Given the equation, $(y-y_1) = m(x-x_1)$, $m=4$ and $x_1=-3$, $y_1=5$. So,
$y-5 = 4(x-(-3)) \Rightarrow y-5 = 4(x+3) \Rightarrow y-5 = 4x+12$
$\Rightarrow y = 4x+17$.

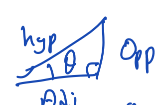
6. Fill in the chart below:

x	$h(x)$	$w(x)$	$(h+w)(x)$	$(h\circ w)(x) = h(w(x))$
0	3	2	$(h+w)(x) = h(x)+w(x)$ $= h(0)+w(0)$ $= 3+2 = 5$	$(h\circ w)(x) = h(w(x))$ $= h(w(0))$ $= h(2) = 6$
1	7	0	$(h+w)(x) = h(x)+w(x)$ $= h(1)+w(1) = 7+0$ $= 7$.	$(h\circ w)(x) = h(w(x))$ $= h(w(1))$ $= h(0) = 3$.
2	6	1	$(h+w)(x) = h(x)+w(x)$ $= h(2)+w(2)$ $= 6+1 = 7$.	$(h\circ w)(x) = h(w(x))$ $= h(w(2))$ $= h(1) = 7$.

We are defining 2 function operations here.
Addition of functions: $(h+w)(x) = h(x)+w(x)$,
and composition of functions: $(h\circ w)(x) = h(w(x))$.

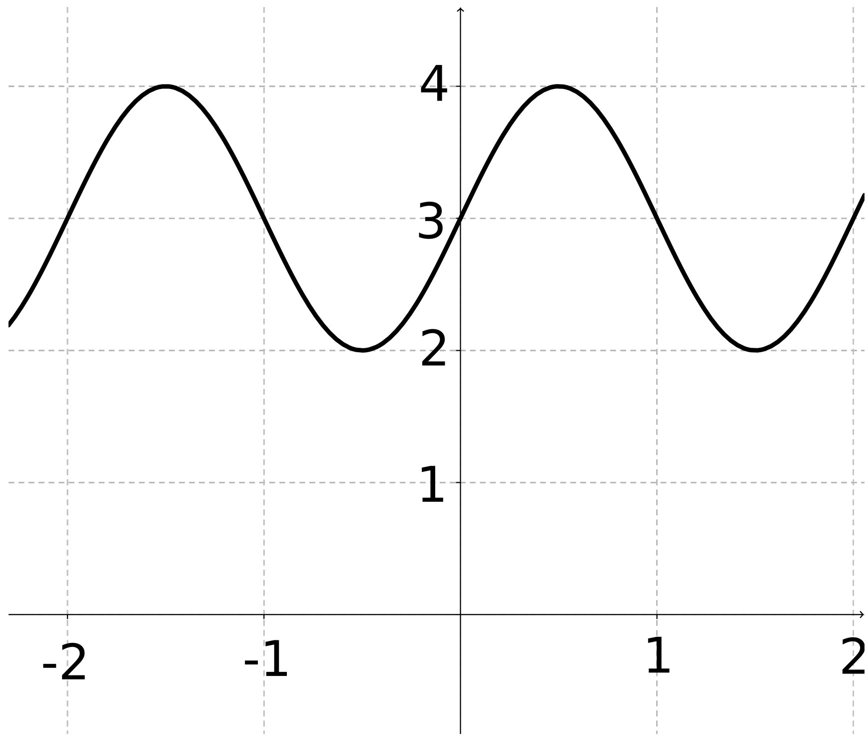
7. A person standing 100 feet away from the base of a building observes that the angle between level ground and the top of the building is 50°. How tall is the building?

• Remember the 3 trig formulas: Given a triangle



∘ $\sin(\theta) = \frac{Opp}{hyp}$
∘ $\cos(\theta) = \frac{Adj}{hyp}$
∘ $\tan(\theta) = \frac{Opp}{Adj}$

• In this case, we have the adjacent side with respect to the angle given, 50°. We want to find the height of the building, which is opp the 50°. So, we use $\tan\theta = \frac{opp}{adj} \Rightarrow \tan(50°) = \frac{Opp}{100} \Rightarrow \therefore Opp = 100\tan(50°)$.



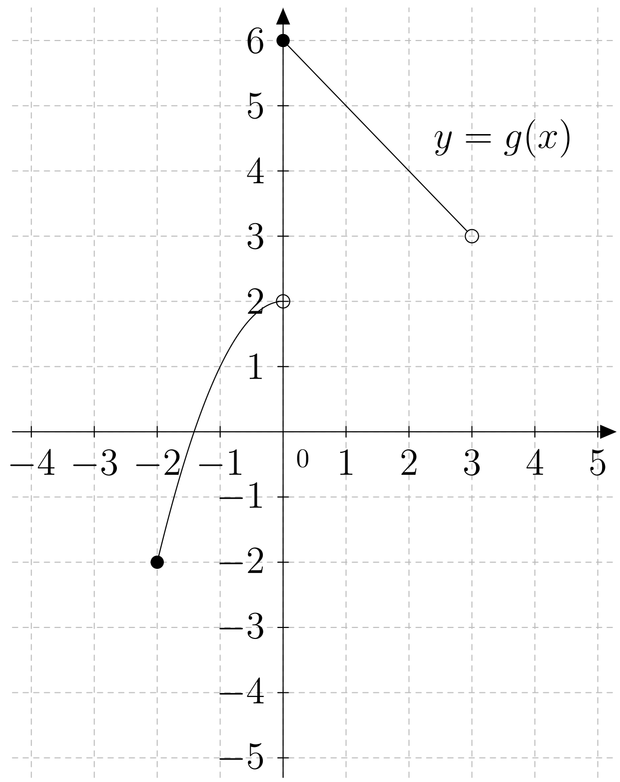
8. For some choice of the parameters a, b, c, and d, the function $y = a\sin(bx+c)+d$ is plotted above. Assume that $a > 0$ and $b > 0$.

A. Find d. → To find d, we want to make $\sin(bx+c) = 0$.

B. Find a.

C. Find b.

D. Find c.



9. Above is the graph of $y = g(x)$.

A. What is the domain of $g(x)$?

↳ the x-values that $y = g(x)$ is defined for. So, $[-2, 3)$.

B. What is the range of $g(x)$?

↳ the set of y-values of $g(x)$. So, $[-2, 2)$ and $(3, 6]$.

C. Where is $g(x)$ increasing?

D. Where is $g(x)$ decreasing?

As the x value increases from -2 to 0, $g(x)$ value increases. So, it increases in the domain $[-2, 0)$.
However, as x increases from $[0, 3)$, the y-value decreases. So, in $[0, 3)$, $g(x)$ is decreasing.

10. Find the roots of the polynomial $2x^2 - 5x - 3$.

Formula for finding roots given a polynomial equation $ax^2 + bx + c$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

So, applying this formula, $x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(-3)(2)}}{2(2)} = \frac{5 \pm \sqrt{25+24}}{4}$
$= \frac{5 \pm 7}{4}$ #.