Name:

Recitation Instructor:

Recitation Time:

Homework #1 is due at 5:00 PM on Jan. 23 in your *recitation's* homework box near Cardwell 120.

1. Write an equation for the line with slope -3 that goes through the point (1, 2).

Slope. $\Leftrightarrow$ Gradient $= -3$

$(x_1, y_1) = (1, 2)$.

Equation of line with gradient m, passing through the point (x_1, y_1) is given by:

$$y - y_1 = m(x - x_1).$$

$\therefore$ we get $y - 2 = (-3)(x-1) \Rightarrow y - 2 = -3x + 3 \Rightarrow y = -3x + 3 + 2 \Rightarrow y = -3x + 5$ #.

2. Let $C(p)$ denote the cost of producing a book with p pages. Suppose that each page added to a book increases its production cost by \$.02 and that a 100 page book costs \$6 to produce. Find a formula for $C(p)$.

$C(p)$ is the cost of producing a book with p pages.

So, $C(p) = A + \$0.02p$, where A is the fixed price regardless of number of pages.

Based on the question, $C(100) = A + \$0.02 \times 100 = \$6$.

$\therefore A + \$2 = \$6$.

$\therefore A = \$4$.

So, $C(p) = \$4 + \$0.02p$.

3. Find all solutions to the equation $\cos(2z + \pi) = 0$.

When we have $\cos(\theta) = 0$,

$\theta = \cos^{-1}(0)$.

$\therefore \theta = \frac{\pi}{2}$

But note that when $\theta = \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \ldots$, $\cos(\theta)$ is also 0. $\therefore$ We write this as $\theta = \frac{\pi}{2} + k\pi$, where $k \in \mathbb{Z}$.

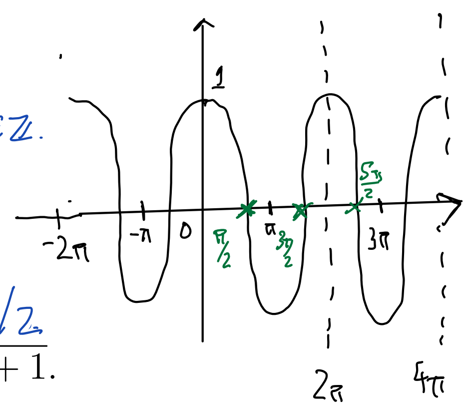
$\cos(2z + \pi) = 0$.

$\Rightarrow 2z + \pi = \cos^{-1}(0) = \frac{\pi}{2} + k\pi, k \in \mathbb{Z}$.

$\Rightarrow 2z = \frac{\pi}{2} - \pi + k\pi$

$\Rightarrow 2z = -\frac{\pi}{2} + k\pi$

$\therefore z = (-\pi/2 + k\pi)/2$.



4. Find functions $f(x)$ and $g(x)$ such that $f(g(x)) = \sqrt{x^3 + 1}$.

This means we input x into g first and then f.

Consider $g(x) = x^3 + 1 \wedge f(x) = \sqrt{x}$.

Then, $f(g(x)) = f(x^3+1) = \sqrt{x^3+1}$ #.

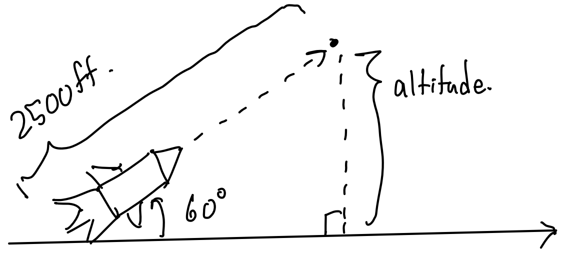
5. If $v(x) = \sin(x)$ and $h(x) = e^x$, find $h(v(x))$.

apply v to x first

$h(v(x)) = h(\sin(x))$ } apply h to sin(x)

$= e^{\sin(x)}$

6. A rocket is fired at sea level and climbs at a constant angle of 60° through a distance of 2500 feet. Find the rocket's altitude.

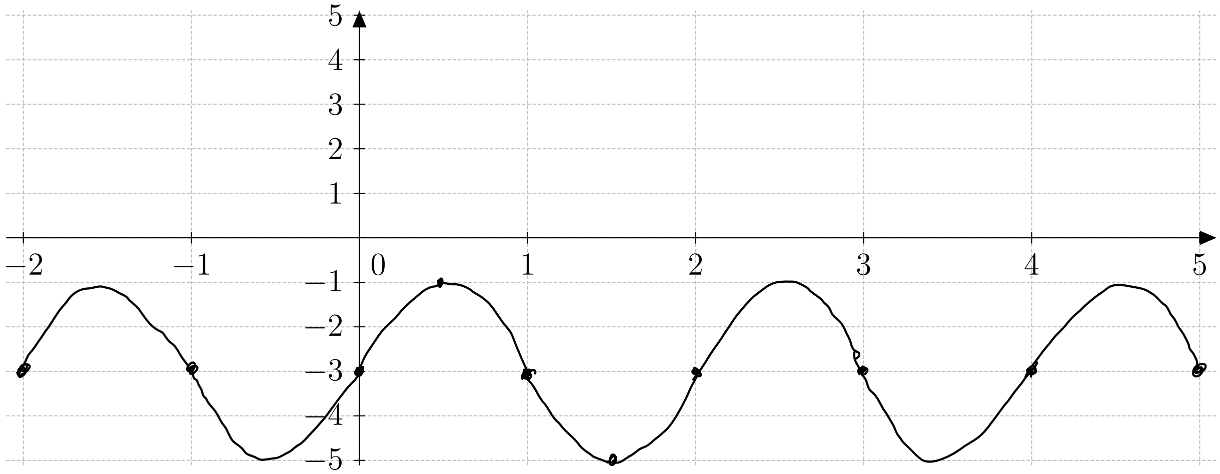


So, with respect to the rocket, the altitude is opposite the angle 60° & the distance 2500ft is opposite the right angle, so it is the hypothenus.

So, we have $\sin(60°) = \dfrac{\text{altitude}}{2500\text{ft}}$.

$\therefore$ altitude $= 2500 \sin(60°)$.

7. Sketch the graph of $y = 2\sin(\pi x) - 3$.

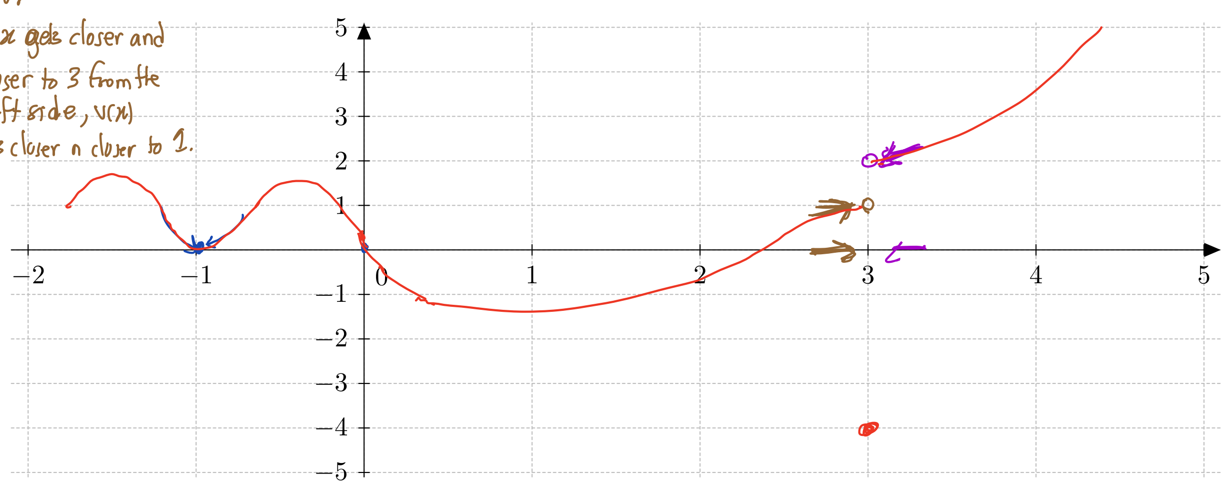


As x gets closer and closer to 3 from the right side, $v(x)$ gets closer and closer to 2

as x gets closer and closer to -1 from both the left & right side, y-values approach 0.

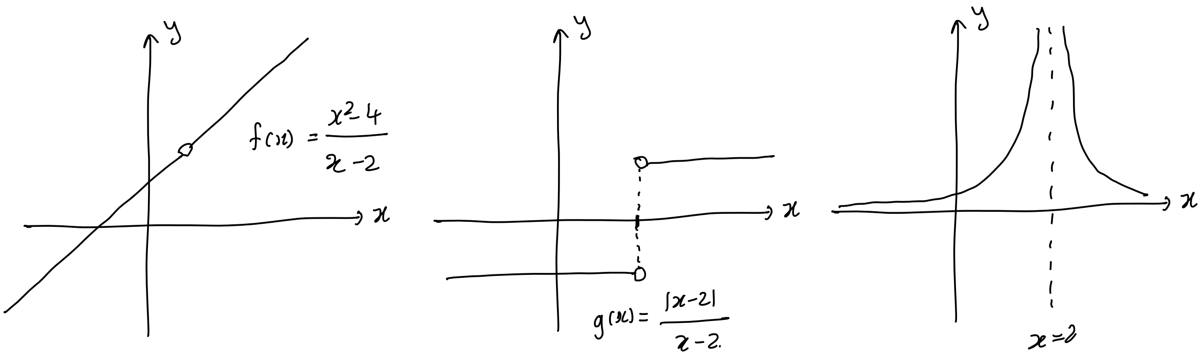
8. Sketch the graph of a function $v(x)$ that satisfies $\lim\limits_{x\to -1} v(x) = 0$, $v(0) = 0$, $\lim\limits_{x\to 3^-} v(x) = 1$, $\lim\limits_{x\to 3^+} v(x) = 2$, and $v(3) = -4$.

as x gets closer and closer to 3 from the left side, $v(x)$ gets closer & closer to 1.

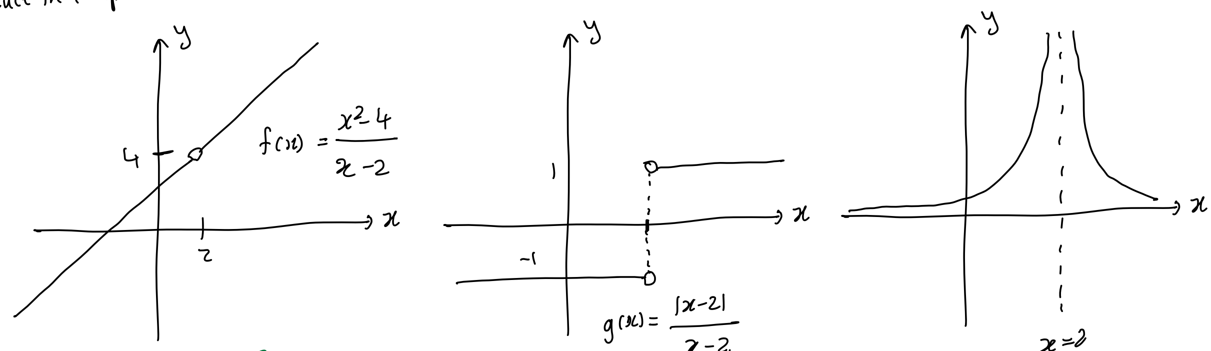


Note that The graph is drawn in red!

• Limits help us understand the behaviour of each graph better. Consider the 3 graphs and focus our attention on the behaviour of each graph at and around $x=2$.



• In each of this function, the function is undefined at $x=2$. But if we make this statement and no other, we give a very incomplete picture of how each function behaves in the vicinity of $x=2$. To express the behaviour of each graph in the vicinity of 2 more completely, we need to introduce the concept of limits.



In this graph, as x approach 2 closer and closer from the left side, we see that the y value gets closer and closer to 4. } left sided/handed limit.

Also, as x approach 2 closer and closer from the right side, we see that the y value gets closer and closer to 4. } Right handed limit

So, we say that $f(x) \to 4$ as $x \to 2$.

As $\lim_{x\to 2^+} f(x) = \lim_{x\to 2^-} f(x)$, we have $\lim_{x\to 2} f(x)$.

Rewrite this as $\lim_{x\to 2} f(x) = 4$.

In this graph, as x approach 2 closer and closer from the left side, we see that the y value gets closer and closer to -1. Whereas, as x approaches 2 closer and closer from the right side, y values get closer and closer to 1.

So, we say that $g(x) \to -1$ as $x \to 2^-$ and $g(x) \to 1$ as $x \to 2^+$.

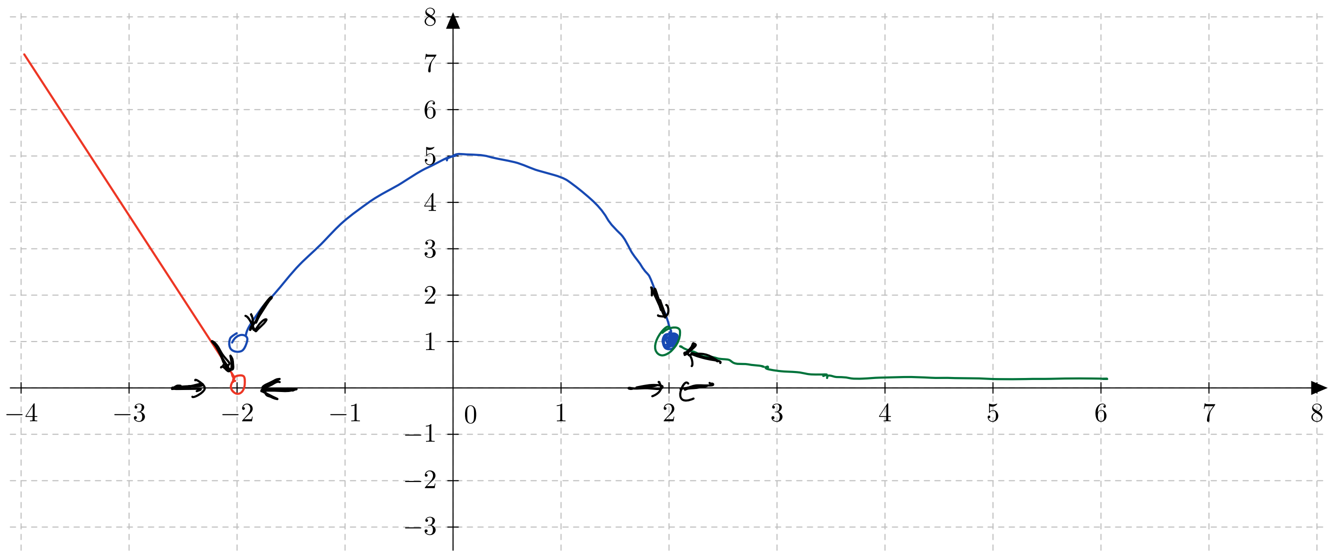
$\therefore \lim_{x\to 2^-} g(x) = -1$ and

$\lim_{x\to 2^+} g(x) = 1$.

Here, as x gets closer and closer to 2 from both the left and right side, y-values approach infinity.

$\therefore \lim_{x\to 2} h(x) = +\infty$.

↳ Also known as vertical asymptote.



9. Graph the function

$$f(x) = \begin{cases} -x-2, & x < -2 \\ -x^2+5, & -2 < x \leq 2 \\ e^{2-x}, & x > 2 \end{cases}$$

in the plot above. Then, use your graph to find the values of each of the below quantities. If the quantity does not exist, write "does not exist".

A. $\lim_{x \to -2^+} f(x)$ ← What value does $f(x)$ approach as $x \to -2$ from the right side? → 1

B. $\lim_{x \to -2^-} f(x)$ ← What value does $f(x)$ approach as $x \to -2$ from the left side? → 0

C. $\lim_{x \to -2} f(x)$ ← What value does $f(x)$ approach as $x \to -2$?
↳ Since $\lim_{x \to -2^-} f(x) \neq \lim_{x \to -2^+} f(x)$, limit DNE.

D. $f(-2)$ → Undefined.

E. $\lim_{x \to 0} f(x)$ ← What value does $f(x)$ approach as $x \to 0$?
$\lim_{x \to 0^+} f(x) = 5 = \lim_{x \to 0^-} f(x)$.
$\therefore \lim f(x) = 5$.

F. $\lim_{x \to 2^-} f(x)$ ← What value does $f(x)$ approach as $x \to 2$ from the left side? → 1

G. $\lim_{x \to 2^+} f(x)$ ← What value does $f(x)$ approach as $x \to 2$ from the right side? → 1.

H. $\lim_{x \to 2} f(x)$ ← What value does $f(x)$ approach as $x \to 2$? → As $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x)$,
$\lim_{x \to 2} f(x) = 1$.

I. $f(2) = 1$.

J. $f(0) = 5$.

$x \to 0$

10. An object dropped from a 100 m tower has height $h(t) = 100 - 4.9t^2$ m for times $0 \text{ s} \le t \le 4.5$ s after being dropped. Recall that over a time interval $[t_1, t_2]$, the *average velocity* of the object is

$$\frac{\Delta h}{\Delta t} = \frac{h(t_2) - h(t_1)}{t_2 - t_1}.$$

A. Find the average velocity from $t = 1$ s to $t = 1.1$ s.

$$\frac{\Delta h}{\Delta t} = \frac{h(1.1) - h(1)}{1.1 - 1} = -10.29$$

B. Find the average velocity from $t = 1$ s to $t = 1.01$ s.

$$\frac{\Delta h}{\Delta t} = \frac{h(1.01) - h(1)}{1.01 - 1} \simeq -9.85$$

C. Find the average velocity from $t = 1$ s to $t = 1.001$ s.

$$\frac{\Delta h}{\Delta t} = \frac{h(1.001) - h(1)}{1.001 - 1} = -9.80$$

D. Estimate the *instantaneous velocity* of the object at time $t = 1$ s.

$\hookrightarrow \Delta t$	Average velocity
0.1	−10.29
0.01	−9.85
0.001	−9.80

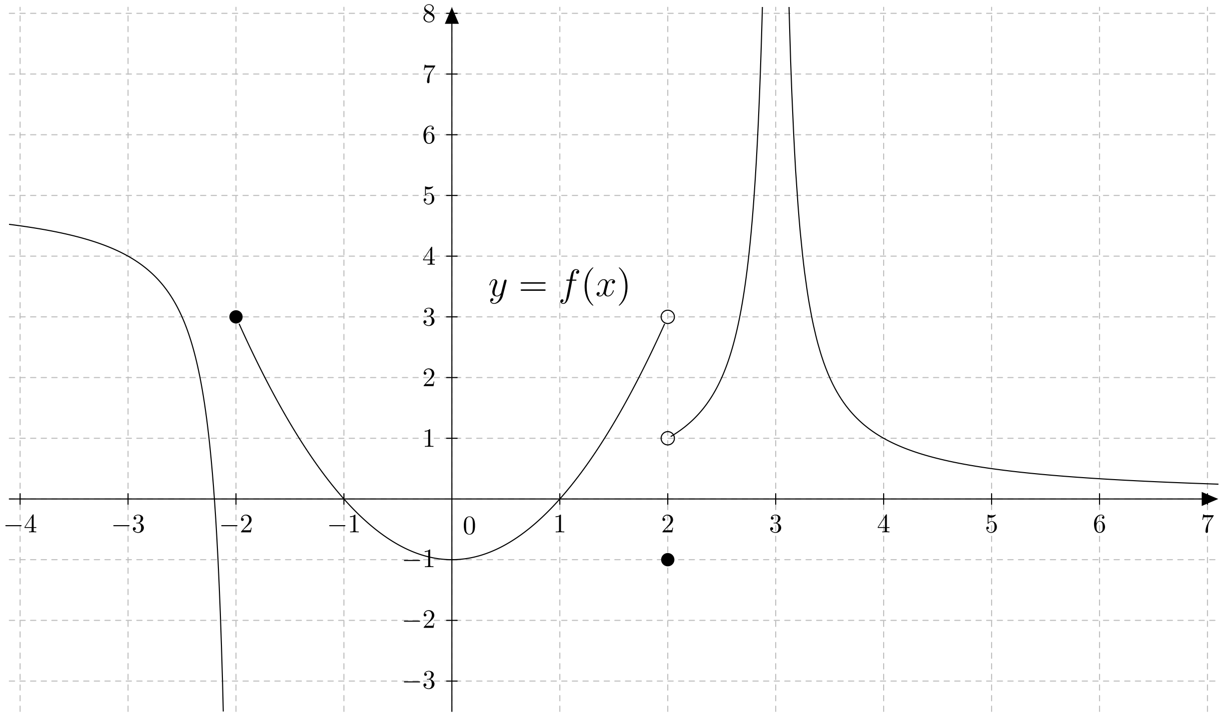
$\therefore$ Instantaneous velocity is estimated to be −9.80 m/s.

11. Let $g(\theta) = \dfrac{\cos(\theta) - 1}{\theta}$. Make a table of values of $g(\theta)$ for $\theta = -.2, -.1, -.01, .01, .1, .2$. (Here, the θ values are in radians.) Use your table to estimate $\lim_{\theta \to 0} g(\theta)$.

θ	$g(\theta)$
−0.2	0.099667
−0.1	0.04958
−0.01	0.0049996

θ	$g(\theta)$
0.2	−0.099667
0.1	−0.049958
0.01	−0.0049999958

$\therefore$ As $\theta \to 0$, $g(\theta) \to 0$.



12. Consider the graph of $y = f(x)$ above. State the value of each of the below quantities. If the quantity does not exist, write "does not exist".

↳ Similar to Qn 9 .

A. $\lim_{x\to 0} f(x)$ → Check what is $\lim_{x\to 0^-} f(x)$ and $\lim_{x\to 0^+} f(x)$. If $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x)$ ⇒ $\lim_{x\to 0} f(x)$ exists.

B. $\lim_{x\to -2^-} f(x)$

C. $\lim_{x\to -2^+} f(x)$

D. $\lim_{x\to 3} f(x)$

E. $\lim_{x\to 2^-} f(x)$

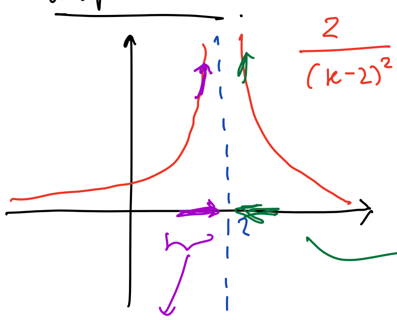
F. $\lim_{x\to 2^+} f(x)$

G. $\lim_{x\to 2} f(x)$

H. $f(2)$

13. Using a graph, table of data, or algebraic reasoning, find $\lim_{x\to 2} \dfrac{2}{(x-2)^2}$.

Graph method



As x approaches 2 from the left hand side, y-values get closer and closer to $+\infty$. $\therefore \lim_{x\to 2^-} \frac{2}{(x-2)^2} = +\infty$.

Similarly, as x gets closer and closer to 2 from the right hand side, y-values get closer and closer to $+\infty$. $\therefore \lim_{x\to 2^+} \frac{2}{(x-2)^2} = +\infty$.

$\therefore \lim_{x\to 2^-} \frac{2}{(x-2)^2} = \lim_{x\to 2^+} \frac{2}{(x+2)^2} = +\infty$.

$\therefore \lim_{x\to 2} \frac{2}{(x-2)^2}$ exist and $\lim_{x\to 2} \frac{2}{(x-2)^2} = +\infty$.