Key concepts

- We covered the idea of limits conceptually and found a few ways to calculate them.

limits form the foundation for our study of Calculus.

Derivatives

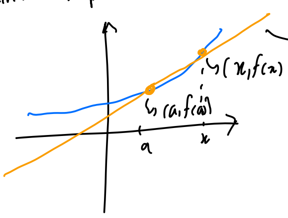
Integrals.

→ Secant & Tangent lines

Def of Secant lines: Given a curve, a secant is a line that passes through any 2 points in the curve.

- Recall that we used the slope of the secant line to a function at a point $(a, f(a))$ to estimate the rate of change, or the rate at which one variable changes in relation to the other.

- We obtain the slope of the secant by choosing a value of x near a and drawing a line through the points $(a, f(a))$ and $(x, f(x))$.

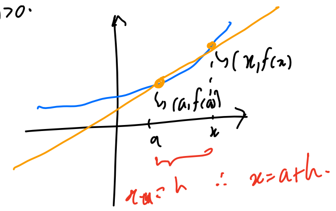


The slope of the secant line through the points : $(a, f(a))$ and $(x, f(x))$ is given by :

$$m_{sec} = \frac{f(x) - f(a)}{x - a}$$

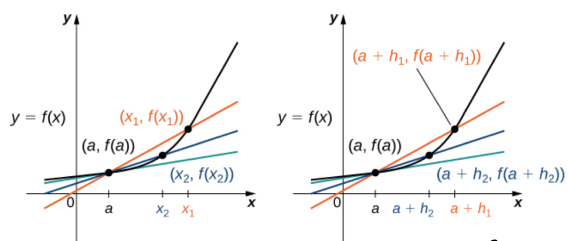
This is called the quotient difference.

- Note that we can also calculate the slope of a secant line to a function at a value a by using the same eqn but expressing x as $a+h$, where $h>0$.



$$\therefore \; m_{sec} = \frac{f(x) - f(a)}{x - a} = \frac{f(a+h) - f(a)}{(a+h) - a} = \frac{f(a+h) - f(a)}{h}.$$

- Note that as $x \to a$, ① The slope of the secant lines provide a better estimate of the rate of change of the function at a.
② The secant lines themselves approach the tangent line to the function at a, which represents the limit of the secant lines.



$$\therefore \; m_{tan} = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}.$$

Definition of tangent line: Let $f(x)$ be a function defined in an open interval containing a. The tangent line to $f(x)$ at a is the line passing through the point $(a, f(a))$ having slope:

$$m_{tan} = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} \quad \text{or} \quad m_{tan} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h},$$

provided the limit exists.

• The type of limit we just computed $m_{tan} = \lim_{x \to a} \frac{f(x)-f(a)}{x-a}$ or $m_{tan} = \lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$ occurs so frequently, that we give this value a special name: The derivative.

Definition of Derivative: Let $f(x)$ be a function defined in an open interval containing a. The derivative of the function $f(x)$ at a, denoted by $f'(a)$, is defined by

$$f'(a) = \lim_{x \to a} \frac{f(x)-f(a)}{x-a} = \lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$$, provided the limit exists.

∴ KEY IDEA : Derivative of $f(x)$ at $x=a$, $f'(a)$ = Slope of tangent line at $x=a$

→ Velocity.

Average velocity: $\frac{s(t)-s(a)}{t-a}$.

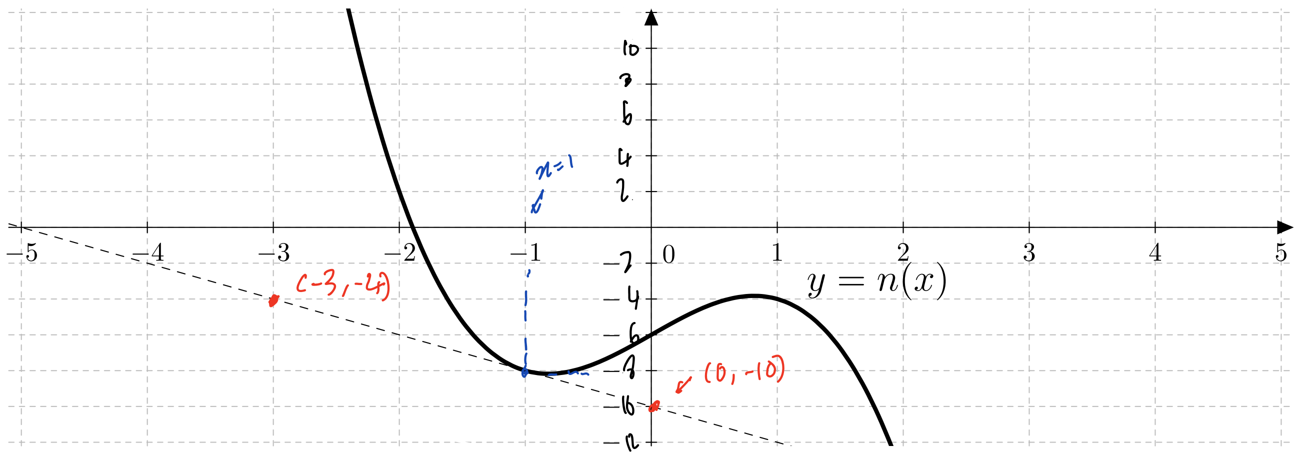
Instantaneous Velocity $= s'(t) = \lim_{t \to a} \frac{s(t)-s(a)}{t-a}$.

Name:

Recitation Instructor:

Recitation Time:

Homework #3 is due at 5:00 PM on **Wednesday**, Feb. 1, in your *recitation's* homework box near Cardwell 120. *This is due on Wednesday so you work the problems before Exam 1!*



1. The function $y = n(x)$ is graphed above in solid bold. There is also a dotted line graphed. Find the following two values.

A. $n(-1) =$ -8

B. $n'(-1) =$

• Remember $n'(-1)$ is the derivative of function $n(x)$ at $x=-1$ and

Derivative of a function $f(x)$ at $x=a$ = Slope of tangent line at $x=a$.

∴ It suffices to find the slope of the tangent line at $x=a$.

$$\frac{-10-(-4)}{0-(-3)} = \frac{-6}{3} = -2.$$

2. Using the limit definition of the derivative, find the derivative of $\frac{1}{x}$ at $x =$ 4.

$f(x) = \frac{1}{x}$ at $x=4$.

$$f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}.$$

Here $f(x)=\frac{1}{x}$ ∧ $a=4$.

$$\therefore f'(a) = \lim_{h\to 0}\frac{f(4+h)-f(4)}{h} = \lim_{h\to 0}\frac{\frac{1}{4+h}-\frac{1}{4}}{h} = \lim_{h\to 0} \frac{1}{4h+h^2} - \frac{1}{4h}$$

$$= \lim_{h\to 0} \frac{4h-4h-h^2}{4h(4h+h^2)}$$

$$= \lim_{h\to 0} \frac{-h^2}{4h(4h+h^2)}$$

$$= \lim_{h\to 0} \frac{-1}{16+4h}$$

$$= -1/16 \ \#.$$

3. Let $w(x) = 5x^2 + 3x + 3$.

A. Using the limit definition of the derivative, find $w'(3)$.

$45+9+3$

$$w'(3) = \lim_{h\to 0} \frac{w(3+h) - w(3)}{h} = \lim_{h\to 0} \frac{5(3+h)^2 + 3(3+h) + 3 - [5\cdot 3^2 + 3\cdot 3 + 3]}{h}$$

$$= \lim_{h\to 0} \frac{5(9+6h+h^2) + 9 + 3h + 3 - 57}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{45} + 30h + 5h^2 + \cancel{12} + 3h - \cancel{57}}{h}$$

$$= \lim_{h\to 0} \frac{30h + 5h^2 + 3h}{h} = \lim_{h\to 0} 30 + 5h + 3$$

$$= 33 \#$$

B. Find the equation for the tangent line to $y = w(x)$ at $x = 3$.

$m_{tan} = f'(3) = 33.$ $\quad f(3) = 57$

$\therefore \ (y - 57) = 33(x - 3)$

4. Using the limit definition of the derivative, find $f'(3)$ given that $f(x) = \sqrt{x+2}$.

$f(x) = \sqrt{x+2}$ at $x = 3$.

$$f'(3) = \lim_{h\to 0} \frac{f(3+h) - f(3)}{h} = \lim_{h\to 0} \frac{\sqrt{3+h+2} - \sqrt{5}}{h}$$

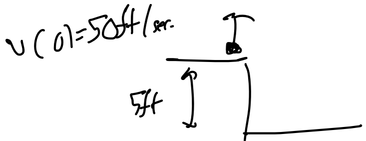
$$= \lim_{h\to 0} \frac{\sqrt{5+h} - \sqrt{5}}{h}$$

$$= \lim_{h\to 0} \frac{(\sqrt{5+h} - \sqrt{5})(\sqrt{5+h} + \sqrt{5})}{h(\sqrt{5+h} + \sqrt{5})}$$

$$= \lim_{h\to 0} \frac{(5+h) - 5}{h(\sqrt{5+h} + \sqrt{5})} = \lim_{h\to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{5+h} + \sqrt{5})}$$

$$= \lim_{h\to 0} \frac{1}{\sqrt{5+h} + \sqrt{5}}$$

$$= \frac{1}{2\sqrt{5}} \#$$

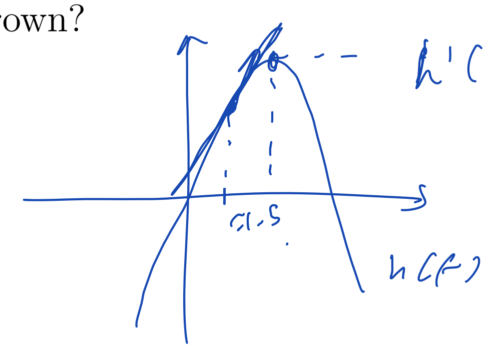


5. When throwing a softball directly upward from a height of 5 ft with an initial velocity of 50 ft/sec, the height of the softball after t seconds is given by $h(t) = -16t^2 + 50t + 5$ (until the ball hits the ground).

A. Using the limit definition of the derivative, find $h'(1)$.

↳ Same as above.

B. Using your answer in Part A, is the ball going upward or going downward one second after being thrown?



$h'(1) > 0.$

$\Rightarrow$ Going upward,